Proceedings of the Jangjeon Mathematical Society www.jangjeon.or.kr
20 (2017), No. 2. pp. 179 – 181 http://dx.doi.org/10.23001/pjms2017.20.2.179

LEFT FILTERS AND PRIME IDEALS ON TERNARYSEMIGROUPS

JAYA LALITHA.G, SARALA.Y, SRINIVASA KUMAR.B,
AND MADHUSUDHANA RAO.D

ABSTRACT. In this article, we give the characterization of a left filter of ternary semigroups. We analyze some relations between the left ternary filters and prime right ideals of a ternary semigroup.

2010 MATHEMATICS SUBJECT CLASSIFICATION. Primary: 06F05; Secondary: 20N99.

KEYWORDS AND PHRASES. Ternary semigroup, Ternary filter, Left(Right, Lateral)ideals, Prime ideals.

1. INTRODUCTION

Kehayopulu [2] gave the characterization of the filter of semigroups interms of the prime ideals. S.K.Lee and S.S .Lee [4] introduced the notion of a left filters on partially ordered semigroups and gave a characterization of the left filters of T interms of prime ideals. Kostaq.H [3] characterized filters in ordered Γ-semigroups. In this paper, we analyze some relations between the left ternary filters and prime right ideals of a ternary semigroup.

2. MAIN RESULT

Definition 2.1. [5] *Let T be a ternary semigroup. A nonempty subset A of T is known as*

(i) a lateral ideal of T if $TAT \subseteq A$.

(ii) a right ideal of T if $ATT \subseteq A$.

(iii) a left ideal of T if $TTA \subseteq A$.

A is known as an ideal of T if it is a lateral, right and left ideal of T.

Definition 2.2. [5] *A subset S of T is known as a prime if $ABC \subseteq S \Rightarrow A \subseteq S$ or $B \subseteq S$ or $C \subseteq S$ for subsets A, B, C of T.*

S is said to be a prime right ideal if S is prime as a right ideal.

S is said to be a prime left ideal if S is prime as a left ideal.

S is said to be a prime lateral ideal if S is prime as a lateral ideal.

S is said to be a prime ideal if S is prime as an ideal.

Definition 2.3. *A subsemigroup F of a ternary semigroup T is known as*

(i) left filter of T if $abc \in F$ for $a, b, c \in T \Rightarrow a \in F$.

(ii) Right filter of T if $bca \in F$ for $a, b, c \in T \Rightarrow a \in F$.

(iii) Lateral filter of T if $bac \in F$ for $a, b, c \in T \Rightarrow a \in F$.

A sub ternary semigroup F of T is known as a filter of T if F is a left, right and lateral filter.

In this paper, we give the characterization of a left filter of T interms of the right prime ideals.

Theorem 2.4. *Let T be a ternary semigroup and F is a nonempty subset of T. The succeeding are equivalent:*
(1) F is a left filter of T.
(2) $T \backslash F = \phi$ or $T \backslash F$ is a prime right ideal.

Proof. (1) $\Rightarrow$ (2): Assume that $T \backslash F \neq \phi$. Let $x \in T \backslash F$ and $y, z \in T$. Then $xyz \in T \backslash F$. Indeed: If $xyz \notin T \backslash F$ then $xyz \in F$. Since F is a left filter, $x \in F$. It is impossible. Thus $xyz \in T \backslash F$ and so $(T \backslash F)TT \subseteq T \backslash F$. Therefore $T \backslash F$ is a right ideal.

Next we shall prove that $T \backslash F$ is prime. Let $xyz \in T \backslash F$ for $x, y, z \in T$. Suppose that $x \notin T \backslash F$; $y \notin T \backslash F$ and $z \notin T \backslash F$. Then $x \in F$; $y \in F$ and $z \in F$. Since F is a subsemigroup of T, $xyz \in F$. It is impossible. Thus $x \in T \backslash F$ or $y \in T \backslash F$ or $z \in T \backslash F$.

Hence $T \backslash F$ is prime and so $T \backslash F$ is a prime right ideal.

(2) $\Rightarrow$ (1): If $T \backslash F = \phi$ then $F = T$. Thus F is a left filter of T.

Next suppose that $T \backslash F$ is a prime right ideal of T. Then F is a subsemigroup of T. Indeed: Assume that $xyz \notin F$ for $x, y, z \in F$. Then $xyz \in T \backslash F$ for $x, y, z \in F$. Since $T \backslash F$ is prime, $x, y, z \in T \backslash F$. It is impossible. Thus $xyz \in F$ and so F is a subsemigroup of T.

Let $xyz \in F$ for $x, y, z \in T$. Then $x \in F$. Indeed: If $x \notin F$, then $x \in T \backslash F$. Since $T \backslash F$ is a prime right ideal of T, $xyz \in (T \backslash F)TT \subseteq T \backslash F$. It is impossible. Thus $x \in F$. Therefore F is a left filter of T. By the similar method, we have the following theorem 2.5.

□

Theorem 2.5. *Let T be a ternary semigroup and F is a nonempty subset of T. The succeeding are equivalent:*
(1) F is a right filter of T.
(2) $T \backslash F = \phi$ or $T \backslash F$ is a prime left ideal.

From Theorem 2.4 and 2.5, we get the following corollary.

Corollary 2.6. *Let T be a ternary semigroup and F is a nonempty subset of T. The succeeding are equivalent:*
(1) F is a filter of T.
(2) $T \backslash F = \phi$ or $T \backslash F$ is a prime ideal of T.

Proof. (1) $\Rightarrow$ (2): Let $T \backslash F \neq \phi$. Then $T \backslash F \neq \phi$ is a prime ideal of T. infact: Since $T \backslash F \neq \phi$, we take a, $b \in T$, $c \in T \backslash F$. If $abc \in F$, then since F is a filter of T, we have $a \in F$, $b \in F$ and $c \in F$. It is impossible. Thus we have $a^3 \in T \backslash F$. i.e $TT(T \backslash F) \subseteq T \backslash F$. Similarly we get $(T \backslash F)TT \subseteq T \backslash F$ and $T(T \backslash F)T \subseteq T \backslash F$. Therefore $T \backslash F$ is an ideal of T. Moreover, Let a, b, $c \in T$ and $abc \in T \backslash F$.

If $a \in F$, $b \in F$ and $c \in F$ then since F is a sub ternary semigroup of T, $abc \in F$. It is impossible. Hence we have $a \in T \backslash F$ or $b \in T \backslash F$ or $c \in T \backslash F$.

(2) $\Rightarrow$(1): Let $T \backslash F = \phi$. Since $T = F$; F is a filter of T. Suppose $T \backslash F$ is a prime ideal of T. Then F is a sub ternary semigroup of T. Infact: a, b, $c \in F$. If $abc \in T \backslash F$, since $T \backslash F$ is prime, $a \in T \backslash F$ or $b \in T \backslash F$ or $c \in T \backslash F$. It is impossible. Thus we have $abc \in F$. Let $a, b, c \in T$ and $abc \in F$. If $a \in T \backslash F$ then, since $T \backslash F$ is an ideal of T, $abc \in T \backslash F$. It is impossible. Thus we have $a \in F$, $b \in F$ and $c \in F$.

Therefore F is a filter of T.

□

References

[1] Anjaneyulu.A; *structure and ideal theory of semigroups, Thesis ANU (1980).*

[2] Kehayopulu.N, *On filters generalized in Po-semigroups,* Math. Japon, **35**(1990),No-4, 789-796.

[3] Kostaq Hila, *Filters in ordered* Γ*-semigroups,* Rocky Mountain Journal of Mathematics, **41**(2011),No-1, 189-203.

[4] Lee.S.K and Lee S.S, *Left(Right) filters on po-semigroups,* Kangweon-Kyungki Mathematics Journal, **8**(2000), No-1, 43-45.

[5] Sarala.Y, Anjaneyulu.A, Madhusudhanarao.D , *Ideals in ternary semigroups,* International e-Journal of Mathematics and Engineering, **203**(2013),1950-1968.

Jaya lalitha.G; Department of Mathematics, K.L.University, Guntur Dt., A.P., INDIA.

E-mail address: jayalalitha.yerrapothu@gmail.com

Dr.sarala.yella; M.sc,M.phill,ph.D; Department of Mathematics, National Institute of Technology, A.P., INDIA.

E-mail address: saralayella1970@gmail.com

Dr.Srinivasa Kumar.B; Department of Mathematics, K.L.University, Guntur Dt., A.P., INDIA.

Dr.Madhusudhana Rao.D; Department of Mathematics, V.S.R and N.V.R College,Guntur Dt., INDIA.